# UC-Lab Center for Distribution System Cybersecurity

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March 2019

# Energy management schemes in distribution networks

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- Design control and market operation algorithms to coordinate smart loads and distribution generation

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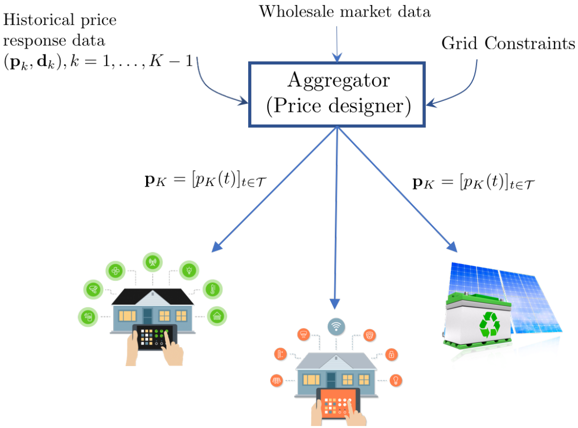
Let us first do a high-level review of demand response architectures

# Architecture #1: Centralized real-time pricing algorithms

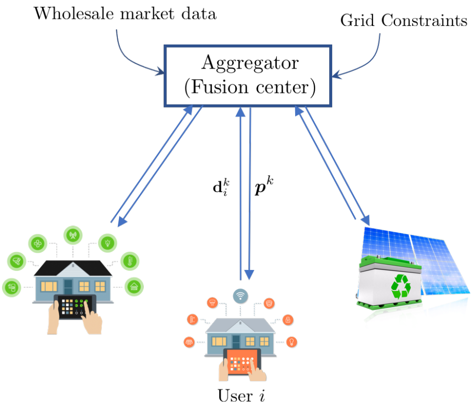
- At the beginning of each day, aggregator posts hourly varying prices
- Essentially an open loop strategy.




Price design based on learned price response from past interactions

# Architecture #2: Distributed coordination algorithms

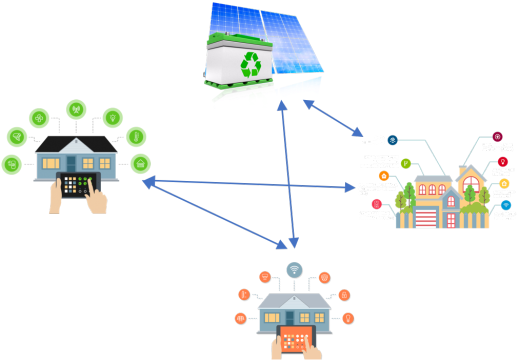
- Decentralized solutions allow users to coordinate to maximize welfare
- Aggregator acts as fusion center for decentralized algorithms




Based on Lagrangian dual decomposition alg $\rightarrow$ Pull demand and push updated electricity price (dual iterates) until convergence

# Architecture #3: Decentralized coordinated algorithms

- No fusion center, only communication with neighbors.



Uses consensus protocols to estimate optimal market clearning prices based on local computation and collaborative message passing

Let us focus on our progress in the **centralized scheme** first

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Suprisingly, this is not as well-studied as decentralized and distributed schemes

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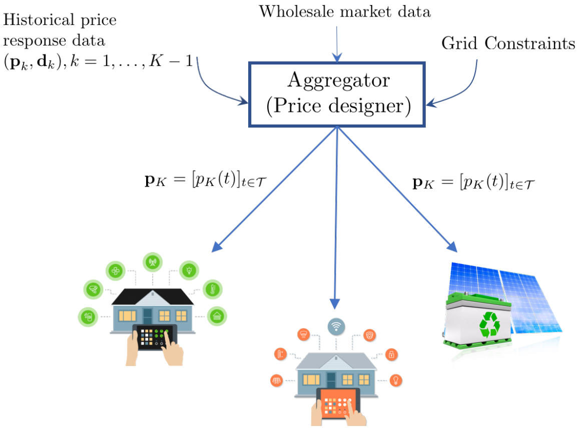


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# Architecture #1: Real-time pricing algorithms

**Main Question**

How to learn the price response of a population of customers to minimize running costs of an aggregator without endangering the distribution grid?

Challenges:

1. Stochastic and unknown nature of customer behavior
2. Variable daily aggregator cost due to changing conditions
3. Small size of the observation

# Problem setup

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**Aggregator's goal?**

$$\min_{\mathbf{p}_t} \sum_{t=1}^{T} g(\boldsymbol{\ell}^\star(\mathbf{p_t}), \mathbf{d_t})$$

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How do we model the load response to prices $\boldsymbol{\ell}^\star(\mathbf{p}_t)$?

$$\mathbb{E}[\boldsymbol{\ell}^\star(\mathbf{p}_t)] = \sum_{i=1}^{Q} a_i(\boldsymbol{\theta}, \mathbf{p}_t)\boldsymbol{\ell}_i^\star(\mathbf{p}_t)$$

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So based on the physical characteristics of the problem,
we have reduced the challenge to not knowing the parameter $\boldsymbol{\theta}$

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- Goal: Maximize payoff over the $N$ limited plays $\rightarrow$ The strategy should not be focused solely on finding the highest paying machine
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- Exploration-Exploitation tradeoff
- Two well-known heuristics with performance guarantees: UCB and Thompson Sampling (confidence region vs. probability distribution)

# Real-time pricing based on multi-armed bandits

**Aggregator's problem**

$$\min_{\mathbf{p}_t} \; \mathbb{E}\left[\sum_{t=1}^{T} g(\boldsymbol{\ell}^\star(\mathbf{p_t}) + \text{noise}, \mathbf{d_t})\right]$$

s.t. grid safety constraints

where

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A. Moradipari, C. Silva, and M. Alizadeh, "Learning to Dynamically Price Electricity Demand Based on Multi-Armed Bandits"
(Thompson sampling performance without any safety constraints)

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- We sample, in each round, a parameter $\boldsymbol{\theta}_t$ from this distribution
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- We observe a noisy version of the load response

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Question: How do we ensure grid safety?

# Reliability-aware MAB formulations

**Aggregator's problem**

$$\min_{\mathbf{p}_t} \sum_{t=1}^{T} g(\boldsymbol{\ell}^\star(\mathbf{p_t}), \mathbf{d_t})$$

$$\text{s.t.} \quad \mathbf{h}(\boldsymbol{\ell}^\star(\mathbf{p}_t), \mathbf{d}_t) \leq \mathbf{0} \qquad \text{(dist flow constraints)}$$

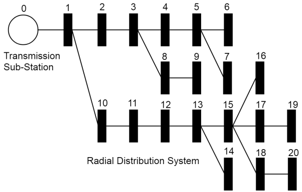
where

$$\boldsymbol{\ell}^\star(\mathbf{p}_t) = \sum_{i=1}^{Q} a_i(\boldsymbol{\theta}, \mathbf{p}_t)\boldsymbol{\ell}_i^\star(\mathbf{p}_t)$$

Potential approaches to ensure safety:

- Lagrangify the constraint (relaxation)
- Ensure the constraints hold with high probability

# Numerical experiment




$\ell^P_{i,t}$ : Active power demand at node $i$

$\ell^Q_{i,t}$ : Reactive power demand at node $i$

$s^P_{i,t}$ : Active power generation at node $i$

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$f^P_{i,t}$ : Active power flow on line $i$

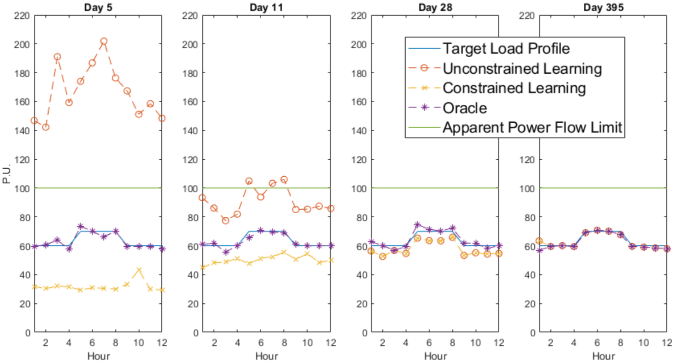
$f^Q_{i,t}$ : Reactive power flow on line $i$

$S^{max}_i$ : Apparent Power Limit of line $i$

$n_f$ : Likelihood of power flow constraint violations

$\mathbf{P}[(f^P_{i,t})^2 + (f^Q_{i,t})^2 \leq (S^{max}_i)^2] \geq 1 - n_f$

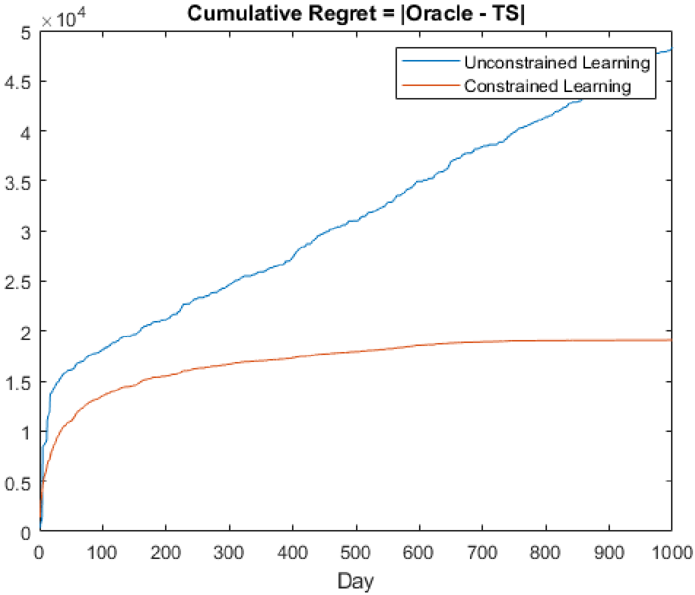
# Numerical experiment: case 1




Apparent Power Line Flow Limit violation ratios:

- Constrained case: 0.0017
- Unconstrained case: 0.1177

# Numerical experiment: case 2



Will discuss how this helps security in future work presentation

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Let's briefly look at the **distributed scheme** now
Ramtin will discuss the decentralized scheme

# Economic distpatch of distributed energy resources

- Individual agents are selfish price takers. They maximize payoff.
- Price responsive demand:

$$\max_{\mathbf{d}_j} \; U_j(\mathbf{d}_j) - \mathbf{p}^T \mathbf{d}_j$$

- Generators:

$$\max_{\mathbf{g}_v} \; \mathbf{p}^T \mathbf{g}_v - C_v(\mathbf{g}_v)$$

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**Welfare maximization (economic dispatch) problem**

$$\begin{aligned} \max_{\mathbf{d},\mathbf{g}} \; & \sum_{j \in \mathcal{J}} U_j(\mathbf{d}_j) - \sum_{v \in \mathcal{V}} C_v(\mathbf{g}_v) \\ \text{s.t.} \; & \mathbf{1}^T \mathbf{d} = \mathbf{1}^T \mathbf{g} \qquad \text{(demand supply balance)} \\ & \mathbf{H}(\mathbf{d} - \mathbf{g}) \preceq \mathbf{c} \quad \text{(line capacity)} \end{aligned}$$

Constraints can be replaced with convexified distribution OPF models

# Market prices

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Why would selfish users follow the solution of this optimization problem?

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## "Pricing" the constraints → Locational Marginal Prices

Based on the Lagrange multipliers of the first and second constraint, we can define the market clearing prices at each bus that maximize welfare:

$$\mathbf{p} = \gamma \mathbf{1} + \mathbf{H}^T \boldsymbol{\mu}$$

# Market prices

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But no single entity knows $U_j(d_j)$ (and potentially $C_v(g_v)$)! So what should we do?

# Decentralized calculation of prices

## Welfare maximization (economic dispatch) problem

$$
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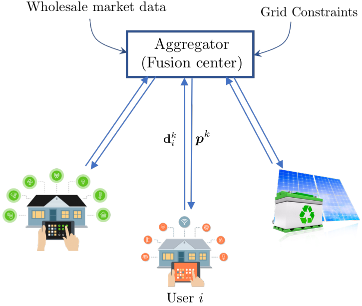
In dual decomposition the constraints are "priced" and then the dual problem is solved $\rightarrow$ dual problem solved via decentralized schemes



# Distributed calculation of prices

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### Dual-decomposition based approach:

- Fusion center updates $\mathbf{p}^{(k)} = \gamma^{(k)}\mathbf{1} + \mathbf{H}^T\boldsymbol{\mu}^{(k)}$ (say using dual subgradient methods)
- Given prices, each individual user solves:

$$\max_{\mathbf{d}_j^{(k)}} U_j(\mathbf{d}_j^{(k)}) - \mathbf{p}^T\mathbf{d}_j^{(k)}, \qquad \max_{\mathbf{g}_v^{(k)}} \mathbf{p}^{(k)\,T}\mathbf{g}_v^{(k)} - C_v(\mathbf{g}_v^{(k)})$$

and shares $\mathbf{d}_j^{(k)}$ and $\mathbf{g}_v^{(k)}$ with fusion center

# The attack surface for distributed schemes

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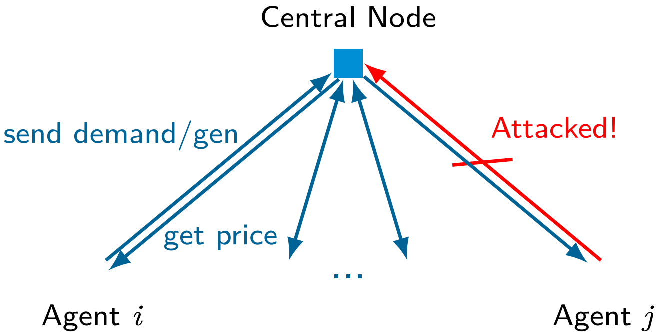
# Attack-resilient network utitlity maximization?



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Assume some of the $d_i$ iterates are adverserially chosen by an attacker in the following classical problem:

$$\begin{aligned}
&\min_{d_i \in \mathbb{R}^d, \forall i} \frac{1}{N} \sum_{i=1}^{N} U_i(d_i) \\
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**The trick**

Apply recent techniques from robust statistics to estimate the correct mean for unaffected agents in the presence of byzantine attacks.

# Overall idea

- Under Byzantine attack $\rightarrow$ impossible to optimize the original problem since the contribution from attacked agents becomes unknown to the fusion center $\rightarrow$ Focus only on trustworth agents:

$$\begin{aligned} \min_{d_i \in \mathbb{R}^d, \forall i \in \mathcal{H}} \ & \frac{1}{|\mathcal{H}|} \sum_{i \in \mathcal{H}} U_i(d_i) \\ \text{s.t.} \quad & g_t\left(\frac{1}{N}\sum_{i=1}^{N} d_i\right) \leq 0, \ t = 1, ..., T, \\ & d_i \in \mathcal{C}_i, \ \forall i \in \mathcal{H}. \end{aligned}$$

Note that the identity of the trustworthy agents ($\mathcal{H}$) are unknown

We will show that the robustified distributed method converges geometrically to a neighborhood of the optimal solution, where the radius of the neighborhood is proportional to the fraction of affected agents

Thank you!